

## Multipath Channel Model using DSP Techniques

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Multipath channels occur in both wireless and wired systems. A model of the multipath channel is often critical for evaluating receiver designs. This article shows how a complex-coefficient FIR filter can be used to model the channel.

The multipath channel is described by a main path of amplitude 1.0, plus one or more paths which each have a unique delay, amplitude, and phase with respect to the main path. The channel may be represented by the following transfer function:

$$H(\omega) = 1 + b_1 e^{-j\omega T_1} + b_2 e^{-j\omega T_2} + \dots$$

where the  $b_i$  are complex and  $T_i$  are the delays of each path with respect to the main path.

Now consider a complex-baseband discrete-time model of the channel, as shown in Figure 1. For this model, the transfer function is evaluated only over the bandwidth of the channel. Substituting  $z = e^{j\omega T_s}$  into the above equation, we get:

$$H(z) = 1 + b_1 z^{-T_1/T_s} + b_2 z^{-T_2/T_s} + \dots$$

If the sample time is sufficiently small compared to the delays, we can round  $T_i/T_s$  to integers  $k, m, \dots$  to obtain:

$$H(z) = 1 + b_1 z^{-k} + b_2 z^{-m} + \dots$$

Clearly,  $H(z)$  can be implemented as a complex-coefficient FIR filter. The number of taps depends on the longest delay  $T_{\max}$  and the sample time:  $N_{\text{taps}} = T_{\max}/T_s + 1$ . Only those taps that match one of the path delays receive a non-zero coefficient. For a two-path model, all taps but the first and last are zero.

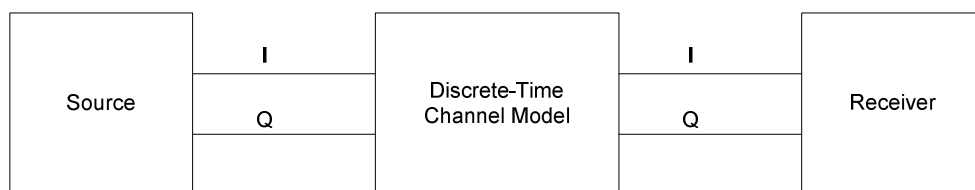


Figure 1. Complex-Baseband Discrete-time Channel

## Example Multipath Channel Model

This is a two-path model that we will evaluate in the frequency domain and in the time domain. The choice of phase angle for the second path is typically not known, so several values are normally evaluated. We choose an arbitrary value here. Let

$$T_1 = 300 \text{ ns}$$
$$b_1 = Ge^{j\varphi}, \text{ where } G = 0.3 \text{ and } \varphi = 0.7\pi$$
$$f_s = 16 \text{ MHz}$$

The number of taps needed in the complex FIR filter is  $\text{round}(T_1 * f_s) + 1 = 6$ , and the delay between the main path and second path is 5 taps. The model's actual delay in the second path is thus  $5/16 \text{ MHz} = 312.5 \text{ ns}$ . The frequency response within the channel bandwidth is calculated using Matlab as follows, with the response magnitude shown in figure 2. The response is asymmetrical because  $H(z)$  is complex.

```
fs = 16; % sample frequency MHz
N = 256;
k = -N/2:N/2-1;
f = k*fs/N;
G = 0.3; phi = 0.7*pi; % path 2 gain and phase

b = [1 0 0 0 0 G*exp(j*phi)]; % multipath coefficients

[h,f] = freqz(b,1,f,fs); % response within channel bw

plot(f,20*log10(abs(h))),grid,axis([-2 2 -4 4]);
```

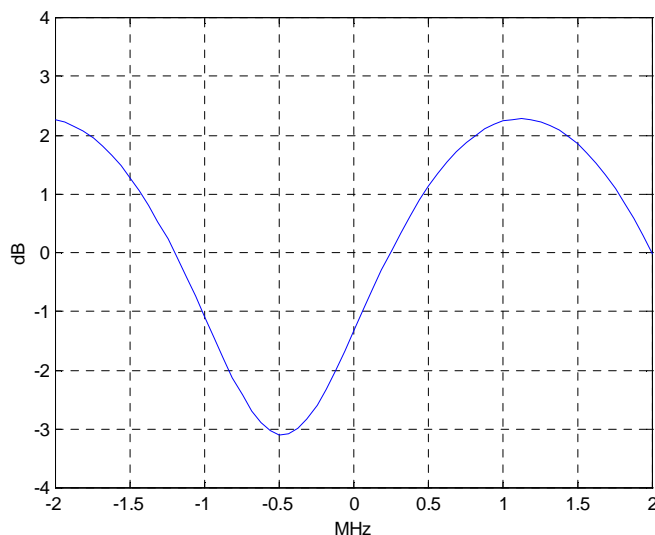


Figure 2. Channel frequency response magnitude within the channel bandwidth for a two-path model.

Now we consider an example time response. The transmitted signal is a real pulse. In other words, the pulse is on I, and  $Q = 0$ . The following code filters the pulse  $x$  with coefficients  $b$ . Figure 3 shows the transmitted pulse, the I and Q components due to the multipath channel, and the envelope of the pulse due to the multipath channel.

```

fs = 16;           % sample frequency MHz
N = 28;
n = 0:N-1;
t = n/fs;
G = 0.3; phi = 0.7*pi; % path 2 gain and phase

b = [1 0 0 0 0 G*exp(j*phi)]; % multipath coefficients

x= [0 .031 .187 .5 .813 .969 1 1 1 1 .969 .813 .5 .187 .031 0 ...
    zeros(1,12)]; % real pulse

y= filter(b,1,x); % pulse after multipath (complex)

z = abs(y); % pulse envelope

```

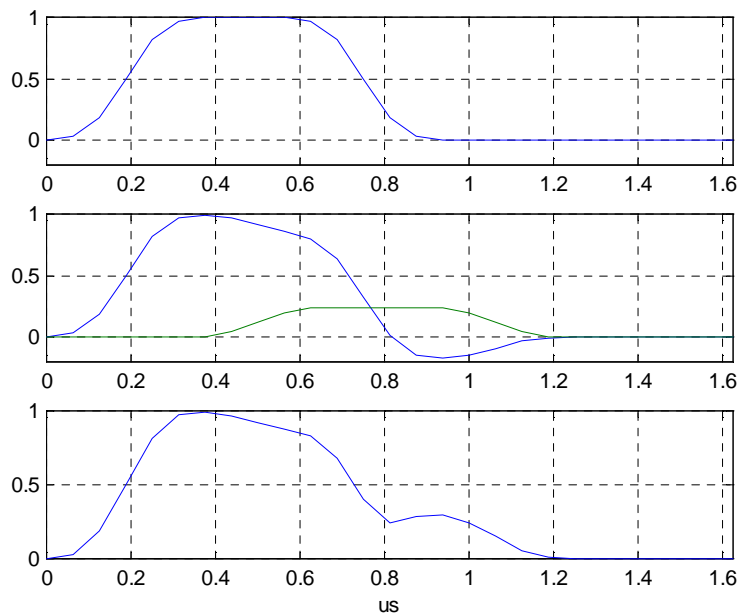


Figure 3. Top: Transmitted real pulse. Middle: I and Q with multipath. Bottom: Pulse envelope with multipath.

## Multipath Channel Model for a QPSK Modulator

Figure 4 shows a complex baseband QPSK Modulator with a complex FIR multipath channel. For this example, the sample rate is 4 times the symbol rate and the multipath channel coefficients are:

$$\mathbf{b} = [1 \ 0 \ 0 \ .3 * e^{j.7 * \pi}]$$

The time delay of the second path is 3 samples/4samples/symbol =  $\frac{3}{4}$  symbol. The constellation exhibits inter-symbol interference due to multipath (Figure 5).

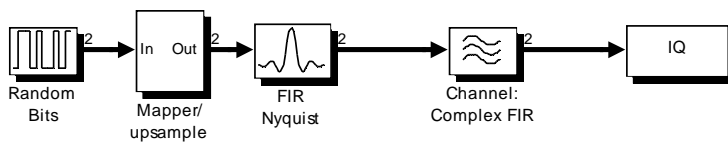


Figure 4. QPSK Modulator with Multipath channel.

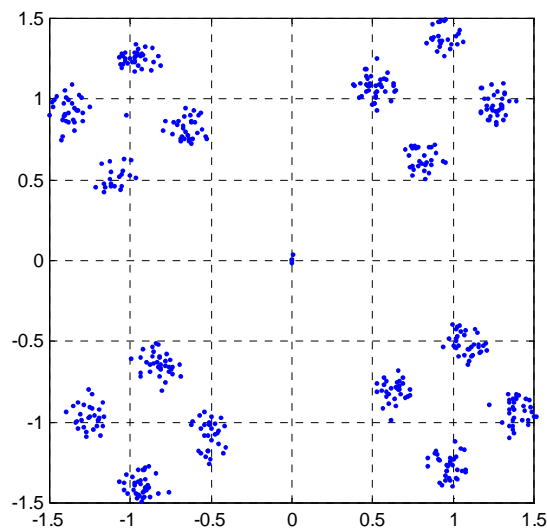


Figure 5. Constellation of QPSK signal after multipath channel.